The phenomenon of **vortex breakdown**

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Abstract: Vortex breakdown is an abrupt change of flow structure that occurs in swirling flows. More than 3 decades of research in this area has not shed light upon this very basic fluid dynamical phenomenon. The mechanism that leads to vortex breakdown is very poorly understood despite numerous theoretical and experimental investigations. There is still a need for a coherent theory. In this paper these issues are considered in the form of a review of current literature on the phenomenon of vortex breakdown. Some existing criteria to predict the breakdown are presented.

1. Introduction

Also known as vortex bursting, the phenomenon of vortex breakdown occurs sometimes in vortex filaments such as trailing vortices. Benjamin (1962) describes this phenomenon as an abrupt and drastic change of structure which sometimes occurs in a swirling flow. Even after three decades of research, however, the physical mechanism that leads to vortex breakdown is not well understood. Vortex breakdown occurs in two distinct types: (1) bubble or axisymmetric type and (2) spiral type (see Fig. 1). In the first type of breakdown, there is a rapid expansion of the core forming a bubble-like structure that is nearly axisymmetric. In the second type, the vortex centerline deforms into a spiral without any appreciable growth in core size. An important feature of vortex breakdown is that the axial flow, which is necessary for the breakdown, decelerates along the vortex axis. In particular, the overall direction of axial flow is usually reversed inside the bubble. Photographic evidence shows that the bubble type of breakdown is dominant in trailing vortices while both forms occur in leading edge vortices over delta wings. Within the breakdown structure the vortex is weaker in the sense that the velocity gradients are smaller since the circulation is redistributed over a larger area. Although this does not imply decay of vorticity in itself, the vortex does decay rather rapidly downstream of breakdown in both types since the flow almost always becomes turbulent. Detailed reviews on vortex breakdown are found in Hall (1972), Leibovich (1978,1984) and Escudier (1988).

2. Experimental studies

Vortex breakdown was first recognized in the aerodynamic context of flow over a highly swept wing (such as a delta wing) at high angle of attack. Early experimental work is reported in Peckham and Atkinson (1957), Elle (1960) and Lambourne and Bryer (1961). Both spiral and bubble types of breakdown were observed in these experiments. Harvey (1962) was among the first to investigate the phenomenon in a controlled laboratory experiment in which the swirl angle, defined as the inverse tangent of the ratio of maximum axial velocity to the maximum swirl velocity, could be varied. Harvey observed onset of a bubble type of vortex breakdown at a 'critical' value of the swirl angle, which he estimated at about 50.5°. This agrees well with the predictions of a theory due to Squire (1960), which estimates this 'critical' value to lie between 45° and 50.2°. The bubble was found to remain stable as the swirl angle was increased. It was also observed that the bubble shape was nearly axisymmetric and the flow downstream of the bubble nearly resembled the parallel flow upstream. This indicates that vortex breakdown is not a result of instability as has been suggested by some researchers (see Hall 1972).

Faler (1976) (also see Faler and Leibovich 1977) has made elaborate studies including flow visualization and quantitative measurements in a circular pipe with a divergence angle of 1.43°. He observed several flow configurations over a range of the two parameters, Reynolds number Re and circulation number N_c, defined as

$$\mathrm{Re} = \frac{\bar{u}D}{\nu}, \qquad N_c = \frac{\Gamma_0}{\bar{u}D} \tag{1}$$

where u is the average axial velocity, D is the inlet diameter of the test section, ν is the kinematic viscosity and Γ_0 is the total circulation. Faler observed seven distinct structures (numbered from 0 to 6), including the usual bubble (type 0) and spiral types (type 2), and a double helix (type 5). Other types are variants of these.

By and large, Faler presents a rather complex picture of the breakdown phenomenon. The breakdown structure is observed to change abruptly from one type to the other and move randomly (though by small distances) in the axial direction. The type 0 breakdown has a bubble whose internal structure consists of two distinct recirculation regions and four stagnation points, as shown in Fig. 2. However, some trends are observed in the dependence of mean axial location of breakdown on Re and N_c as shown in Fig. 3. In this figure, r denotes inlet test section radius and $x/r = 0$ corresponds to the inlet location. Thus for $N_c > 1.54$, the breakdown structure actually moves upstream of the test section. It is also noted by Faler that for similar values of N_c there are two major differences in his results when compared with the experimental results of Sarpkaya (1971): (1) Breakdown locations differ considerably for the same value of Re, and (2) Reynolds number for transition from spiral type (type 2) to bubble type (type 0) differs greatly. This is attributed by Faler to possible differences in the geometry of the experimental apparatus.

Suematsu *et al.* (1986) observed vortex breakdown in a rotating circular pipe. Their measurements suggest that a stationary internal wave of finite amplitude occurs which ultimately results in an axisymmetric bubble type of breakdown. This wave is seen to emerge when the ratio of maximum swirl velocity to the maximum axial velocity reaches a certain critical value. However, no estimate of this critical value is reported by the authors.

Garg and Leibovich (1979) report that wakes of breakdown flowfields contain prominent oscillations at a frequency less than 10 Hz, experimental conditions being identical to those of Faler (1976) described above. The oscillations are found to be more energetic and the core expansions are larger for bubble type, compared to the spiral type and hence the former is a stronger mo-de of breakdown.

In a set up different from that of Faler (1976), Escudier and Zehnder (1982) observed a variety of disturbances to occur upon a vortex filament. In their experiment, they used an inlet cylindrical tube coupled with a contraction followed often by a divergent sectioa whose angle varies from 3° to 5°. The flow enters the inlet cylindrical tube tangentially at a single point and causes swirling motion. In particular, Escudier and Zehnder recognized that the breakdown depends on three parameters: (1) Reynolds number $\mathrm{Re} = \bar{u}D/\nu$, (2) circulation number $N_c = \Gamma_0/uD$, and (3) velocity ratio $v = Q/\Gamma_0 l_c$, where $\bar{u}$ is the average axial velocity in the tube, Γ_0 is the total circulation, D is the tube diameter, Q is volumetric flow rate and l_c is the length of the inlet cylinder. Their experiment suggests that at breakdown, the critical Reynolds number $\mathrm{Re}_B \sim 1/(N_c^3\bar{v})$. This is summarized * Fig. 4. Escudier and Zehnder observed flow patterns similar to Faler's (1976).

Later in 1984, Escudier reported vortex breakdown in a confined swirling flow-m& apparatus consisting of a closed cylindrical container of height h_c and radius r_c filled with fluid. The fluid was set in motion by rotating one of the endwalls. A boundary lay* that develops on the rotating endwall feeds vorticity into a vortex filament that appears on the symmetry axis. The flow is determined entirely by two parameters: the aspect ratio h_c/r_c and Reynolds number based on rotation $\mathrm{Re} = \Omega_e r_c^2/\nu$, where Ω_e is the angular frequency of the rotating endwall and ν is the kinematic viscosity of the fluid. Depending on these parameters, Escudier observed either no breakdown or up to three breakdown bubbles (with recirculating regions). In some cases, the bubble was seen to oscillate. His results are summarized in Fig. 5. This set up has been studied numerically by Lopez (1990) Fig. 6 shows a comparison. This specific problem is further analyzed by Brown and Lopez (1990) and Lopez and Perry (1992).

The basic features that have emerged from the experiments are: (1) abrupt and drastic structural changes occur in a vortex breakdown, (2) axial flow in the core decelerates, sometimes resulting in stagnation and reversal of flow, (3) the flow is unsteady within the breakdown structure and turbulent downstream, (4) axisymmetric (or bubble) breakdown is characterized by slow oscillations, i.e., flow is nearly steady, (5) the bubble structure is very complex consisting of two recirculating regions and four stagnation points, and (6) the breakdown itself is not a result of instability but a sudden and finite transition from one state to the other (Harvey 1962).

3. Theoretical and numerical studies

Stuart (1987) has classified theoretical work on vortex breakdown into three main categories: (1) theories based on hydrodynamic instability, (2) theories based on deceleration of axial flow leading to stagnation, and (3) theories based on transition from one flow state to another. These three viewpoints are considered below.

Harvey's (1962) experimental evidence strongly suggests that vortex breakdown is not due to instability. Also, Stuart (1987) observes that hydrodynamic instability appears to be insensitive to downstream boundary conditions which is contrary to the observations in a vortex breakdown. For instance, application of downstream suction results in a vortex flow without breakdown. Thus, flow instability as a mechanism is considered unimportant.

Axial deceleration is in fact a natural tendency of rotating flows that are dominantly axisymmetric. To illustrate this, consider steady, incompressible and inviscid flow in a nearly cylindrical vortex. Assume the fluid density to be unity. Let (x, θ, r) denote cylindrical coordinates in the axial, swirl and radial directions respectively. Let velocity components be denoted by (u, v, w). The approximate radial momentum equation for pressure p is given by

$$\frac{dp}{\partial r} = \frac{v^2}{r}. \tag{2}$$

Integrating Eq. (2) we get

$$\frac{dp}{dr}\Big|_{r=0} = \frac{dp}{dx}\Big|_r - \frac{1}{4\pi^2}\int_0^r \frac{\partial \Gamma^2}{dx}\frac{dr}{r^3} - 2\pi v r \tag{3}$$

where Γ is the circulation. Hall (1972) shows that, since Γ is conserved on a streamsurface, equation (3) reduces to

$$\frac{dp}{dx}\Big|_{r=0} = \frac{dp}{dx}\Big|_r + m\,\frac{\Gamma^2}{4\pi^2 r^2} \tag{4}$$

where m is related to the ratio w/u. Upstream of breakdown $m > 0$ since $u > 0$ and $w > 0$. Eq. (4) then implies that in a vortex the axial pressure gradient on the axis is higher than elsewhere. Thus, if an adverse pressure gradient is impressed upon the vortex due to external flow, say, then its effect on the axis is maximum and the flow naturally tends to decelerate. This idea is used by Brown and Lopez (1990) to arrive at a criterion for vortex breakdown (see Section 4).

Benjamin's (1962) theory of conjugate states is based on the idea of criticality which reflects the ability of a given flow to support waves of infinitesimal amplitude. Benjamin explains vortex breakdown to be a finite amplitude transition from a *supercritical* state of flow to a conjugate *subcritical* state. A flow is said to be supercritical if it is unable to support infinitesimal standing waves, while in a subcritical flow standing waves can occur. For a given supercritical state, there exist infinitely many conjugate states, but the one selected in a vortex breakdown is *adjacent* to the given primary supercritical state in a sense to be made clear below.

Assuming that a given primary flow is supercritical (designated as state A), Benjamin has shown that if it undergoes a transition to another cylindrical state (designated as B) the latter is necessarily subcritical. With each state can be associated a 'flow force'

$$F = 2\pi \int_0^b (u_p^2 + p)\, dy, \qquad y = \frac{1}{2} r^2 \qquad (1)$$

which is the integral of the axial momentum flux plus pressure over a section through the flow. Here, u_p is the axial velocity in the primary vortex and $y = b$ denotes the vortex boundary. The flow force associated with the primary supercritical state F_A is less than the flow force F_B of the subcritical state. The state B is adjacent to the state A in the sense that the gain in flow force $F_B - F_A$ is minimum. Any other conjugate state would result in a larger gain. (Fraenkel (1967) has rigorously proved, using a method different from Benjamin's, that to every supercritical flow there exists an adjacent conjugate flow with larger flow force.) The amount of gain, however, determines the type of transition. When the gain is not so large (actual values or estimates are not given by the theory) the resulting breakdown is a mild one in which waves appear on the state B whose 'wave resistance' exactly balances the gain in flow force. This is rigorously proved by Benjamin (1967) using perturbation analysis. When the gain is large, the resulting breakdown is a strong one in which gain in the flow force is balanced by turbulent dissipation. The net result in any case is a development of steady supercritical and subcritical states with the same value of flow force.

Benjamin finds it desirable to introduce the nondimensional number

$$N_{cr} = \frac{c_+ + c_-}{c_+ - c_-} \qquad (2)$$

where c_+ and c_- are the absolute velocities, measured positively in the direction of the flow, at which very long waves propagate with and against the flow respectively. The flow is supercritical if $N_{cr} > 1$ and subcritical if $N_{cr} < 1$. Computing N_{cr} is important for the following reason. If there were no swirl, N_{cr} would be very large and hence the flow always supercritical. Increasing the swirl brings down the value of N_{cr} and breakdown is expected to occur when N_{cr} is very near unity. According to Benjamin, typical values for N_{cr} near breakdown lie between 1 and 2. Similarly, as the axial velocity decelerates, N_{cr} begins to decrease leading to breakdown. Thus, N_{cr} is expected to vary inversely with the swirl parameter S, defined as the ratio v_m/u_m where v_m and u_m are the respective maxima of swirl and axial velocity (see Leibovich and Kribus 1989 for a discussion of the role of S).

Thus, the closest one can get to predicting the breakdown location is to compute the axial location where the axial velocity has retarded sufficiently so that N_{cr} is near unity. However, breakdown may occur for larger N_{cr} if the flow is perturbed sufficiently by means of such agencies as ambient turbulence.

Two limitations of Benjamin's theory may be noted: (1) the theory does not provide any insight into the type of structure within the breakdown region or its extent, and (2) the retardation of axial flow does not appear to be an inherent feature of breakdown.

Beran (1989) investigated solutions of the Navier-Stokes equations for the case of an isolated trailing vortex and for swirling flow through a frictionless pipe. He found the no to be supercritical in the entire computational domain for small swirl parameter S (defined earlier). With increase in 5, the flow is found to become critical at some axial location where a transition point forms. The general features of this transition are found to be in agreement with Benjamin's theory. Flow reversals are observed only for large S. Beran also found that breakdown is not always associated with the formation of a stagnation point on the vortex axis. However, the axial velocity must decelerate for breakdown to occur.

Moore and Saffman (1972) arrived at an equation governing the motion of a vortex filament with arbitrary distribution of swirl and axial velocity within its core. Their method was based on balancing the forces acting on an element of the filament. The forces were determined accurate to second order in a small parameter defined as the ratio of local core radius to local radius of curvature of the filament. In the final simplification, Moore and Saffman dropped the terms containing the variation of core radius along the vortex axis. Retaining these terms and simplifying the equations for an isolated nearly straight vortex, Mudkavi (1991) obtained the following pair of equations

$$\frac{\partial a^2}{dt} + \frac{\partial a^2\overline{u}}{ds} = 0 \tag{7}$$

$$a^2\left(\frac{\partial\overline{u}}{\partial t} + \overline{u}\,\frac{\partial\overline{u}}{\partial s}\right) = -\frac{\partial}{\partial s}\left(\frac{\Gamma^2}{8\pi^2}\ln a^2 + \Gamma^2\varepsilon a^2\right), \tag{8}$$

where a is the core radius, Γ is the vortex strength, t is the time and 5 is arclength coordinate measured along the vortex centerline. The quantity $u(a, s)$ is the axial velocity and an overbar denotes the average over the cross section. The quantity £ is a measure of the deviation of the square of the axial velocity from its average, given by

$$\varepsilon = \frac{1}{\Gamma^2}\left(\overline{u^2} - \overline{u}^2\right). \tag{9}$$

By assuming slug flow within the vortex core (axial velocity is uniform), Lundgren and Ashurst (1989) obtained equations for a straight isolated vortex that are similar to (7) and (8) except that for their case £ is zero. Further, they recognized that these two equations are analogous to those of two-dimensional gas dynamics in which the role 0! density is played by the core radius. Since these equations admit shocks, a sudden transition from one type of flow ('supersonic', equivalent to supercritical flow in the sense of Benjamin) to another ('subsonic' or subcritical) is possible. Across this shock the value of core radius jumps. Thus the model equations (7) and (S) describe essential features of vortex breakdown in accordance with Benjamin's theory. A necessary condition for breakdown predicted by the model equations is that at some axial location the flow must be supercritical, i.e.,

$$\frac{}{v_m} > \sqrt{4\pi^2 a^2\varepsilon + \frac{1}{2}}, \quad v_m = \frac{\Gamma}{2\pi a} \tag{10}$$

where v_m is the swirl velocity at the core radius.

A sample solution of (7) and (8) for a straight vortex with periodic boundary conditions in the axial direction is shown in Fig. 7. This figure shows the cross section of the vortex in the meridional plane. Formation of a jump at a suitable nondimensiorial time $t = 0.6$ is c y visible. Attempts to compare solutions of the above equations with solutions of the full Navier-Stokes equations at large Reynolds numbers by Mudkavi were not successful. Thus the usefulness of model Eqs. (7) and (8) is limited.

There are available in the literature several numerical simulations of vortex breakdown none of which throws any light on the mechanism of breakdown. We list some here in addition to the literature cited above: Grabowski and Berger (1976), Lugt and Haussling (1982), Hafez *et al.* (1986), Nakamura et al (1986), Lugt and Abbound (1987), Lugt and Gorski (1988), Neitzel (1988), Salas and Kuruvila (1989).

4. Vortex breakdown criteria

We briefly review some criteria for vortex breakdown. Brown and Lopez (1990), based on criterion of stagnation on the axis, arrive at a breakdown criterion which states that, for breakdown to occur, the helix angle for velocity must be larger than the helix angle for TOrticity. This only suggests that the streamsurface will diverge. But rapid divergence is not implied.

According to Benjamin (1962, 1967), the nondimensional number N_{cr} defined in (6) must exceed unity for breakdown. For most cases, N_{cr} lies between 1 and 2. Computation of N_{cr} as a function of axial distance on the vortex is nontrivial. It is also not clear if N_{cr} is universal, i.e., independent of the velocity distributions.

Based upon previous theoretical, experimental and computational studies, Spall *et al.* (1987, also see Spall and Gatski 1987) have proposed a different criterion for the onset of vortex breakdown. They observe that for a vortex in an unbounded domain or in a pipe, breakdown is more or less independent of the Reynolds number and depends mainly on the swirl parameter 5. The swirl parameter can be characterized via a Rossby number Ro, defined by Spall *et al.* as

$$\mathrm{Ro} = \frac{u^*}{r^*\Omega^*} \tag{11}$$

where u^*, r^* and fi* represent characteristic velocity, length and rotation frequency respectively. For the wing tip or trailing vortices, they take r* to be the radial distance at which the swirl velocity is maximum. u^* is taken to be the axial velocity at r*. The characteristic frequency Ω^* is taken to be the rotation rate at the vortex centerline. Fig. 8 shows a compilation of results due to theoretical, experimental and numerical investigations of wing tip type vortices as a function of Rossby number and Reynolds number. The Reynolds number for this purpose is defined as r^*u^*/ν where v is the kinematic viscosity of the fluid. Solid symbols stand for flows with breakdown. The solid line marks the breakdown boundary. The results suggest that for Reynolds number above about 100, breakdown occurs if the Rossby number defined by (11) lies below about 0.65.

5. Conclusions

We have reviewed the literature concerning vortex breakdown. While the experimental picture is very clear on the breakdown structure and several numerical studies are able to capture some of these features, an understanding on the mechanism of vortex breakdown is still lacking. There also appears to be no verification of theories with either experiments or computations. There exist only few criteria for the prediction of vortex breakdown. The criterion of Spall *et al.* (1987) appears to be the best method to predict onset of breakdown, but it requires a good estimate of Rossby number. Estimate of the Reynolds number Re is not crucial since the results are independent of Re beyond Re = 100. The location of breakdown is highly unpredictable and its behaviour is random as suggested by some experiments.

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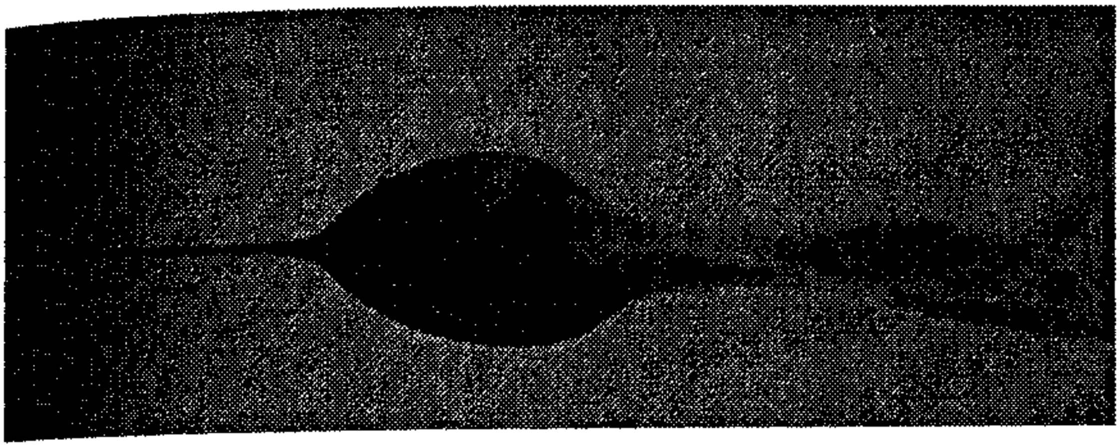
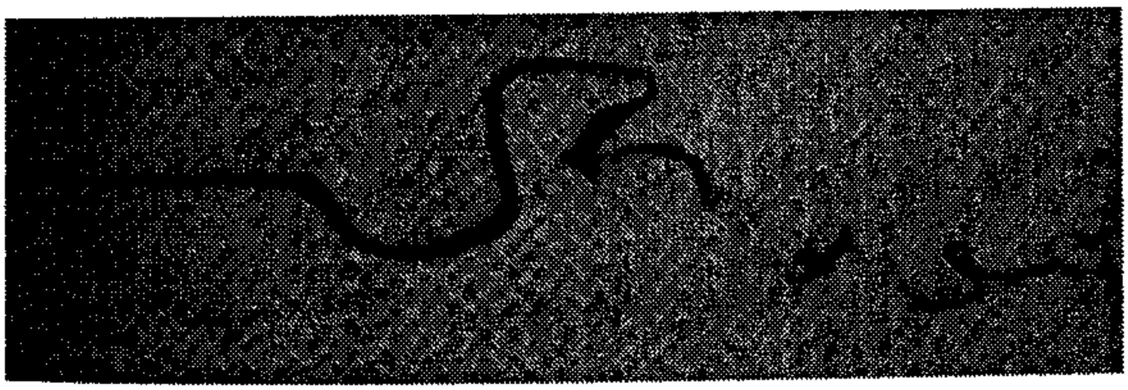
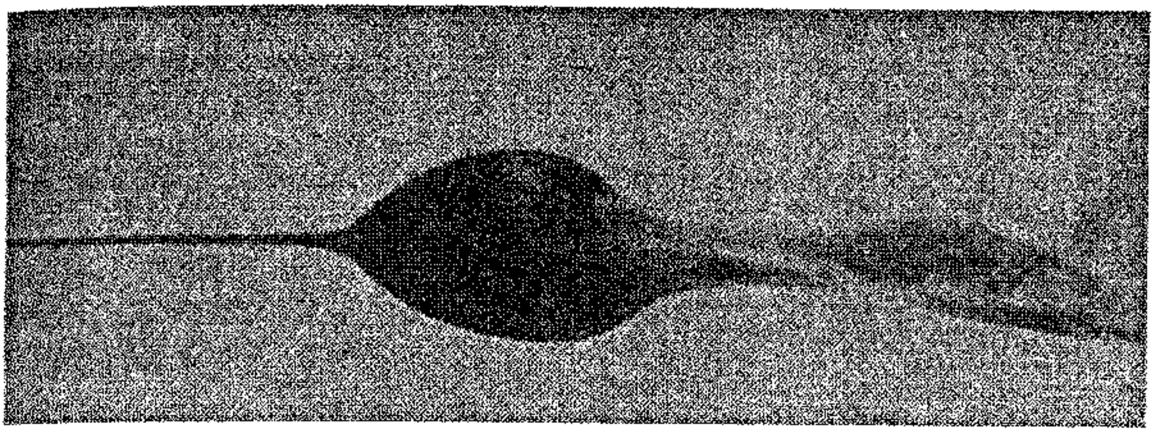


(a) Bubble or axisymmetric type of vortex breakdown. Flow is from left to right. Flow conditions: Re = 2560 and N_c= 1.777.



(b) Spiral type of vortex breakdown. Flow is from left to right. Flow conditions: Re=3120 and N_v =1.34 (these are approximate values quoted based on discussion found in Faler 1976 since exact values for this photograph are not given by the author).

fig. 1 Photographs showing two common types of breakdown observed in swirling flows (Faler 1976).



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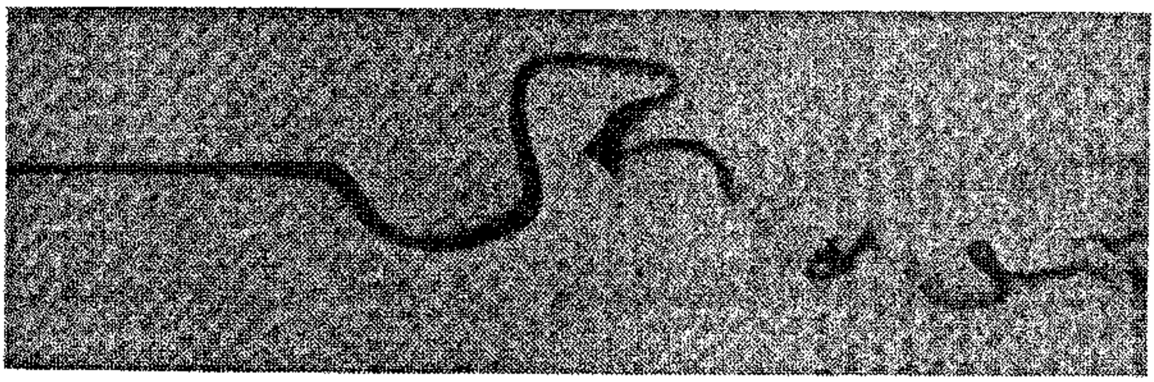

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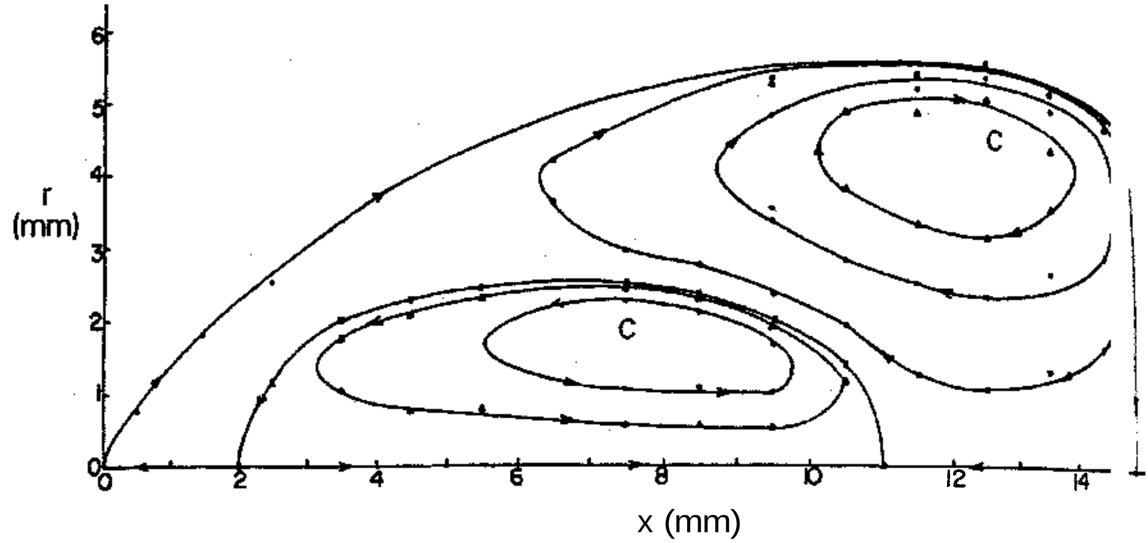

Fig. 2 Time averaged streamlines inside the bubble of axisymmetric breakdown. Symbol C denotes center of a recirculation cell. Other symbols indicate experimental values (Faler 1976).

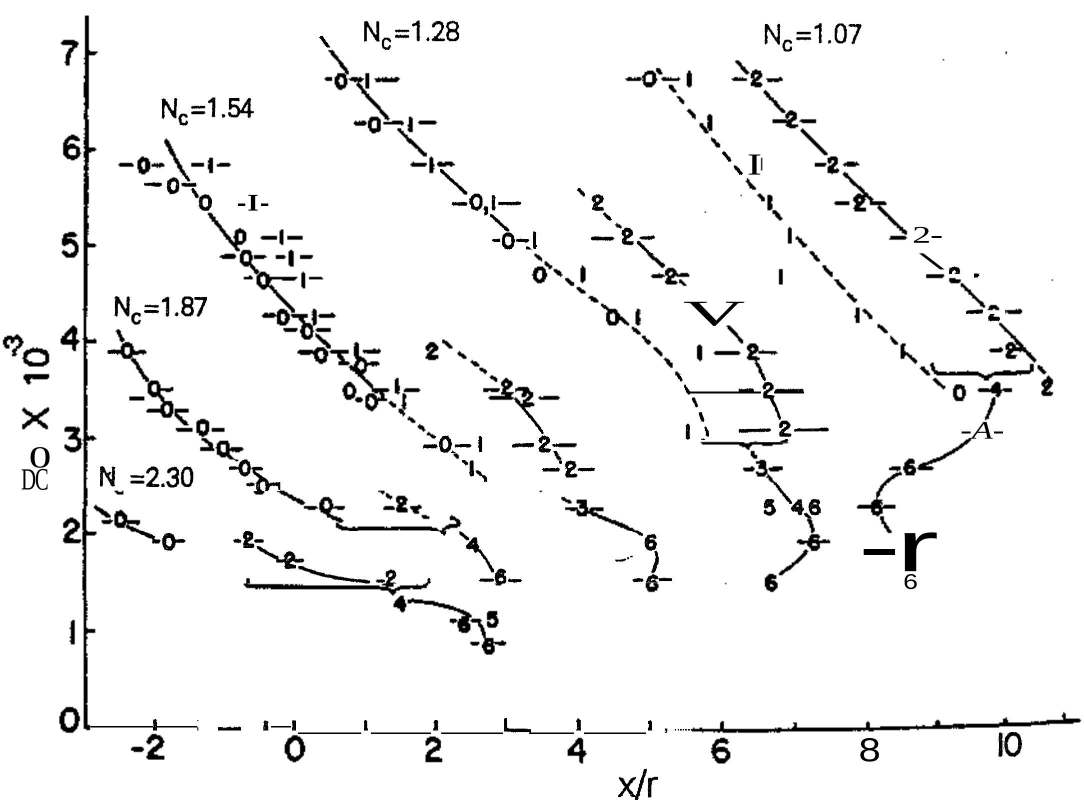

Fig. 3 Mean axial locations of various types of breakdown (types range from 0 to 6 a are indicated by these symbols) as a function of Reynolds number Re for five vates N_c (Faler 1976).

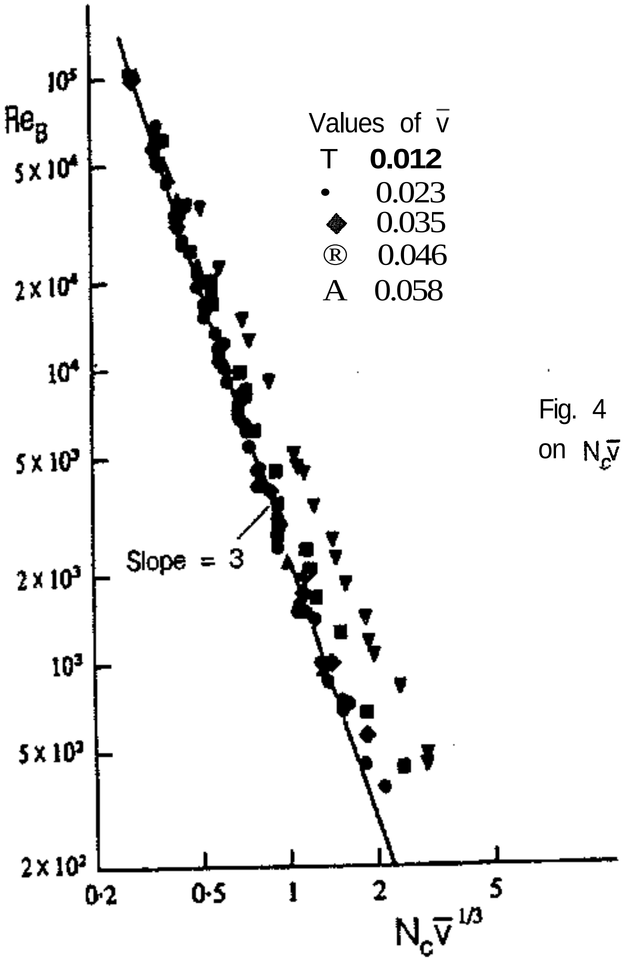

Fig. 4 Dependence of Re for breakdown on $N_c \bar{v}^{1/3}$ (Escudier & Zehnder 1982).

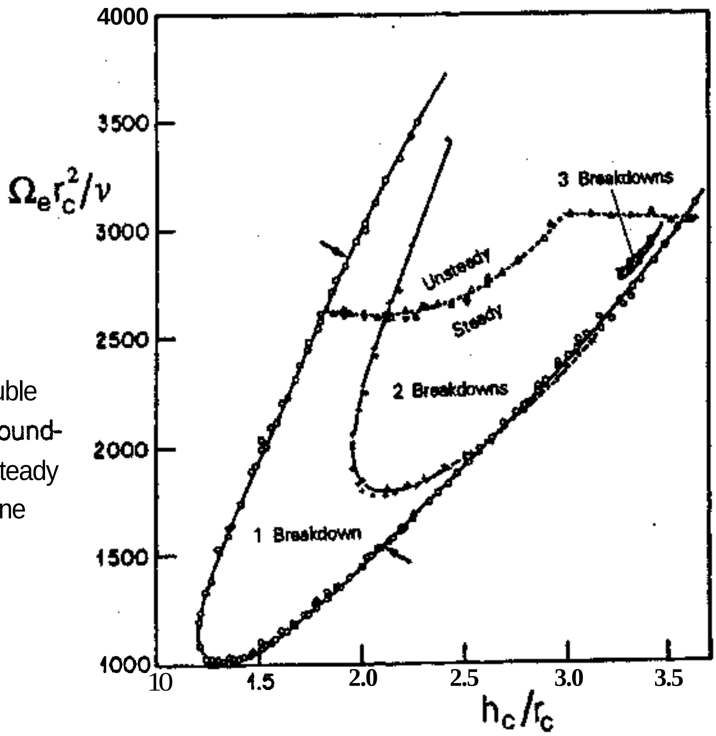

fig. 5 Regions of single, double and triple breakdowns and boundary between steady and unsteady tow in fRe, aspect ratio) plane (Escudier 1984).

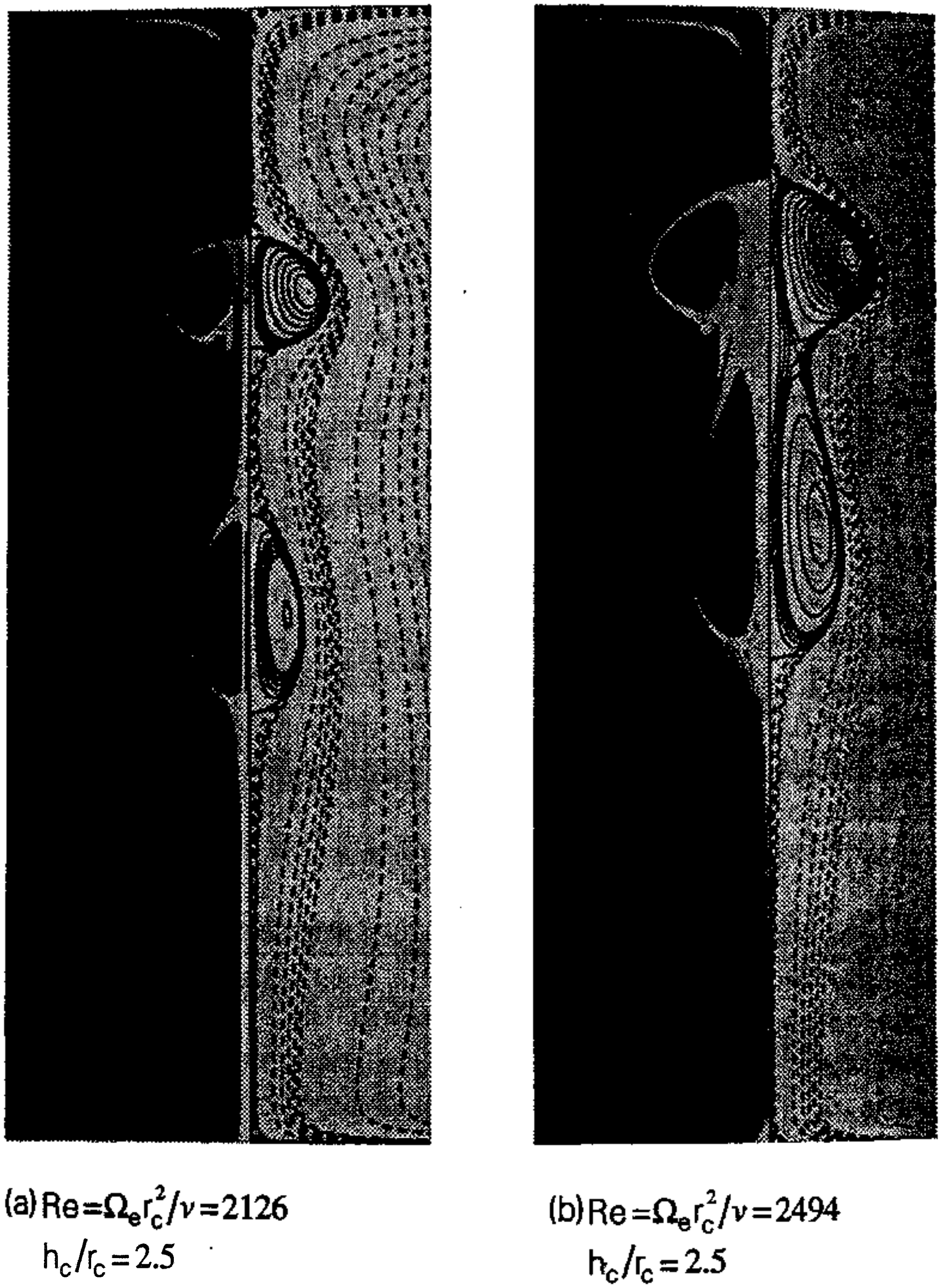

Fig. 6 Comparison of Lopez's (1990) numerical results on the right half with experimental visualizations of Escudier (1984) on the left half of each figure for two different Reynolds numbers. Computed results show streamlines while visualizations show steady dye lines (from Lopez 1990).

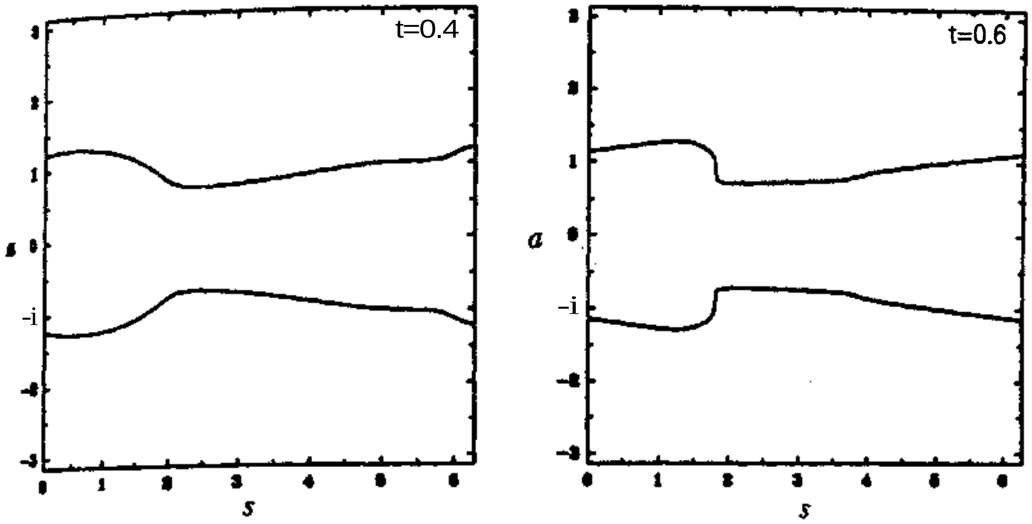

Fig. 7 Numerical solution of equations (6.27) and (6.28). The figure shows the shape of the vortex in longitudinal section at nondimensional times of 0.4 and 0.6 (nondimensionalized on linear wave speed). Periodicity in s is assumed (Mudkavi 1991).

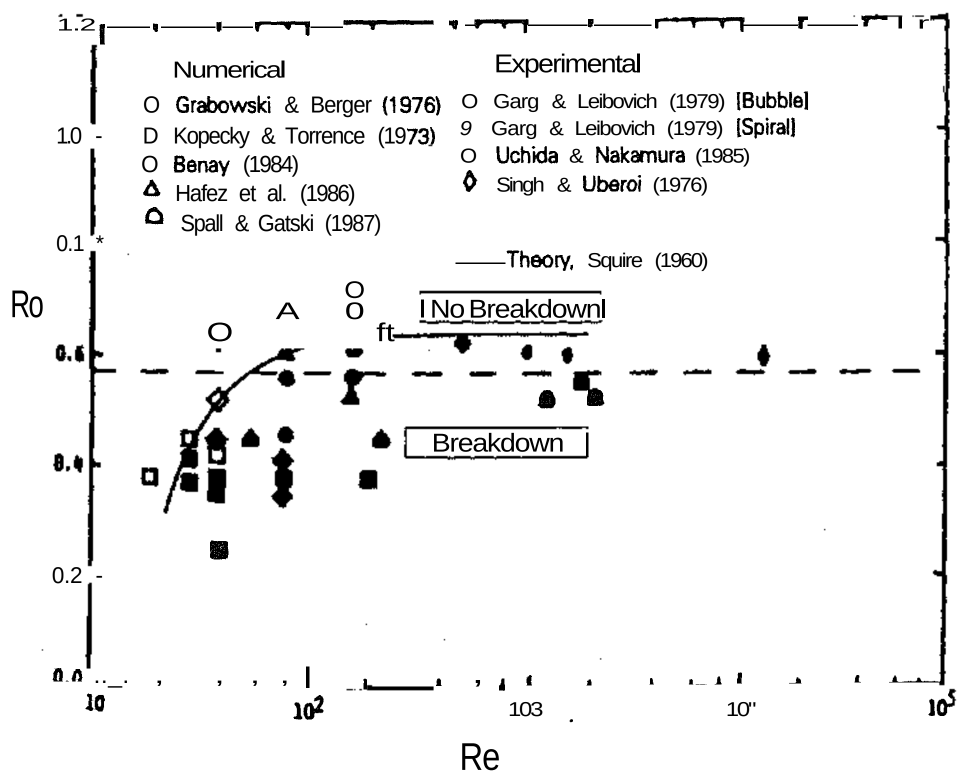

Fig. 8 Relationship between Rossby number Ro and Reynolds number Re for vortex breakdown. The solid line indicates a possible criterion usable for prediction of vortex breakdown (Spall & Gatski 1987).